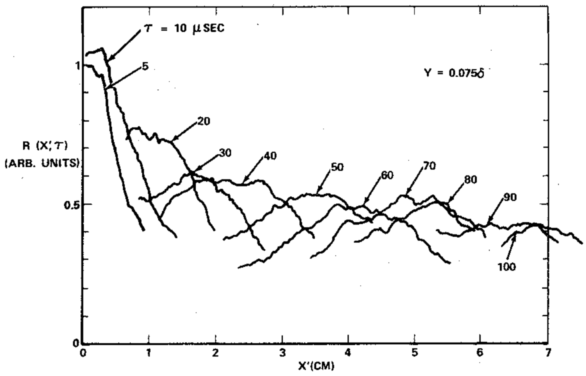

Fig. 2 Typical space-time correlation functions at fixed overheat currents.

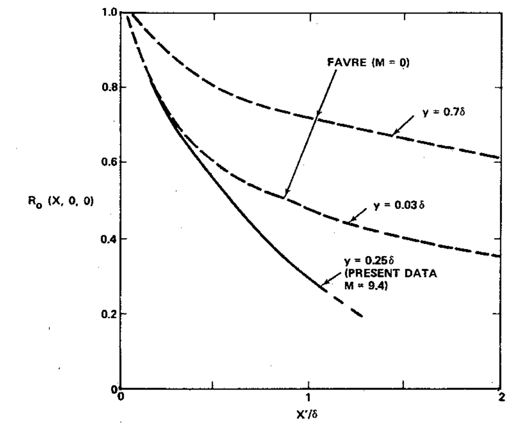

Fig. 3 Optimum longitudinal autocorrelation functions.

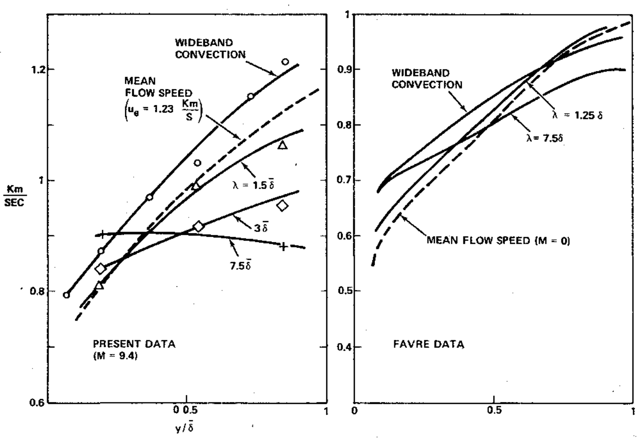

Fig. 4 Eddy convection velocities.

shown for the total (or wideband) turbulence as well as for turbulence filtered to follow eddies of size $\lambda = 1.5\delta$, 3δ, and 7.5δ. The wideband convection speed is slightly higher than the mean flow speed. The filtered correlations have given eddy speeds higher than the mean speed and lower than the mean speed near and away from the wall respectively, the more so as the eddy size increases. Such behavior is qualitatively similar to the lower Mach number case, as shown on Fig. 4; note that for eddy sizes $\lambda \sim 7.5\delta$ the speed is nearly constant at about 900 m/sec or $0.75u_e$. This latter result is in agreement with data from Ref. 2, which, however, also assigns a nearly equal speed to the total (unfiltered) turbulence, in contrast to our findings as stated above.

In summary, certain features of the correlation tensor of the hypersonic turbulent boundary layer seem to depart from the corresponding features of the low-speed boundary layer; in particular, the Mach 9.4 layer studied here decayed faster than expected from low-speed data. The dependence of convection speed on eddy size follows the trend observed in, but is more pronounced than, the low-speed data. What is of concern is the disagreement found between the present data and the hypersonic data of Horstman and Owen,[2] in which the turbulence decay is extremely slow and in which the convection speed of the total, unfiltered turbulence is almost equal to the speed of the large eddies. This may be due to the "transitional" state of the boundary layer of Ref. 2, or even to the small differences in geometry and Mach number between it and the present work. Further experiments at high Mach numbers would therefore seen necessary to shed some light on this issue.

References

[1] Favre, A., Gaviglio, J., and Dumas, R., "Structure of Velocity Space-Time Correlations in a Boundary Layer," *Physics of Fluids,* Vol. 10, Part II, Sept. 1967, pp. 138-145.

[2] Owen, F.K. and Horstman, C.C., "Hypersonic Transitional Boundary Layers," *AIAA Journal*, Vol. 10, June 1972, pp. 769-775.

[3] Laderman, A.J. and Demetriades, A., "Mean and Fluctuating Flow Measurements in the Hypersonic Boundary Layer Over a Cooled Wall," *Journal of Fluid Mechanics*, Vol. 63, Part 1, 1974, pp. 121-144.

[4] Demetriades, A., and Laderman, A.J. "Reynolds Stress Measurements in a Hypersonic Boundary Layer," *AIAA Journal*, Vol. 11, Nov. 1973, pp. 1594-1596.

[5] Demetriades, A., "Modal Analysis of Turbulent Correlations in Compressible Flow," *Transactions of the ASME*, Series E, *Journal of Applied Mechanics,* Vol. 40, Sept. 1973, p. 822-823.

[6] Morkovin, M.V., "Fluctuations and Hot-Wire Anemometry in Compressible Flows," AGARDograph 24, 1956.

Slip Model for Hypersonic Viscous Flow

B. N. Srivastava,* R. T. Davis†
and
M. J. Werle‡
University of Cincinnati, Cincinnati, Ohio

Introduction

SEVERAL studies[1-3] in the recent past have indicated that a continuum flow analysis to estimate the surface properties of hypersonic vehicles with prolonged exposure at high altitudes is sufficient provided that slip effects are accounted for properly. Many authors[4-6] have studied the phenomenological expressions for slip velocity and slip temperature on such a surface. More recent studies[7] have been conducted where these expressions were determined by comparing the results of numerical integration of the Navier-Stokes equations with experimental data. No rigorous theoretical conclusion is possible, but, insofar as agreement with experiment goes, the use of continuum flow analysis with proper slip effects has been found time and again to be quite

Received July 8, 1975; revision received Oct. 16, 1975. This research was supported by the U.S. Air Force through the Arnold Engineering Development Center under Contract F40600-74-C-0011-P00001.

Index categories: Supersonic and Hypersonic Flow; Viscous Non-boundary-Layer Flows; Jets, Wakes, and Viscid-Inviscid Flow Interactions.

*Graduate Research Assistant, Department of Aerospace Engineering.

†Professor and Head, Department of Aerospace Engineering. Member AIAA.

‡Professor, Department of Aerospace Engineering. Member AIAA.

adequate. The purpose of the present study is to demonstrate the influence of the slip conditions and the slip constants on the results predicted by the hypersonic viscous shock layer equations, these being an approximate representation of the compressible counterpart of the Navier-Stokes equations.

Governing Equations and Method of Solution

The equations and notations used are the same as those used by Davis.[8] It is unnecessary to present the equations, since they are included in the paper cited. Two sets of surface conditions were considered in the present study. The first set was the same as that used by Davis [Eq. (2.7), Ref. 8] and will be referred to here as standard slip conditions. A second set of surface conditions as proposed in Ref. 9 was also employed (referred to here as the new slip conditions). These new slip conditions are identical to the standard set except that the shear stress τ in the velocity jump condition [Eq. (2.7b), Ref. 8] is replaced by the expression

$$\Omega = \mu[u_n + \kappa u / (1 + \kappa u)]$$

which is the vorticity on the surface. The conditions at the position of the shock were represented here by a modified form of the Rankine-Hugoniot relations, which are commonly known as "shock slip conditions," the details of which are given in Ref. 8.

The viscous shock layer equations were solved using a method similar to that developed for solving the boundary-layer equations, such as the method of Blottner and Flugge-Lotz.[10] However, in the present problem, the calculation of the shock wave shape at the outer boundary represents an elliptic effect and must be handled in a fashion that reflects the boundary-value nature of the problem. Thus, in the present problem, the global variation of the shock shape over the entire integration region is represented by curve-fitting the shock shape at each stage of the iteration process used here to solve the governing equations. This method is found to represent the shock slope accurately and to lead to converged solutions for the problem considered. For detailed description of the numerical method employed, one may refer to Refs. 11 and 12.

Results and Discussion

Figure 1 gives a comparison of the surface pressure distributions from the present results with the experimental data of Boylan[13] for hypersonic flow past a paraboloid. Attention is directed first to the perfect gas case (constant specific heats and Prandtl number), which was solved under the assumptions that the thermal accomodation coefficient and fraction of incident molecules diffusely reflected were unity. This result is seen to compare well, when the standard slip condition is used, with experimental data everywhere except in the stagnation region, where there is severe error. In order to assess the sensitivity of these results to the perfect gas assumption, these calculations were repeated using a binary mixture of nitrogen atoms and molecules.§ It is observed from Fig. 1 that gas model has virtually no influence on the predicted results, at least under these freestream conditions, and thus cannot be held accountable for the stagnation-region error. This certainly raises some question about the flow model being used, but it also points to a dilemma where it is not clear whether the experimental measurements produce the pressure or the normal stress at the surface. It only can be assumed at this juncture that the quantity provided by the experimental data is the same thermodynamic pressure being calculated in the present solution.

In an effort to test further the slip model itself, the surface pressure distributions were also obtained using the new slip conditions[9] as shown in Fig. 1. It is seen here that under the present test conditions the standard slip conditions and the new slip conditions give identical results. It is believed, as pointed out in Ref. 9, that the new slip conditions would be more pertinent for values of Knudsen number $\gg 1$ or at least of the order of one. The stagnation region error, thus, remains unaccounted. It was found further, through a series of calculations,[11] that the removal of wall slip from the model produced an overcorrection to the stagnation region level of surface pressure, whereas the effect of shock slip, which represents a departure from Rankine-Hugoniot shock conditions, was only of a minor consequence for this test con-

§Viscosity coefficients, specific heats, enthalpies, and the thermal conductivities were the same as given by Davis.[14] Lewis number was assumed to be 1.4.

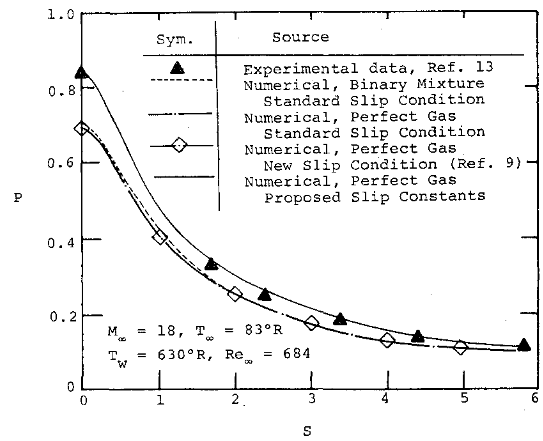

Fig. 1 Surface pressure distribution.

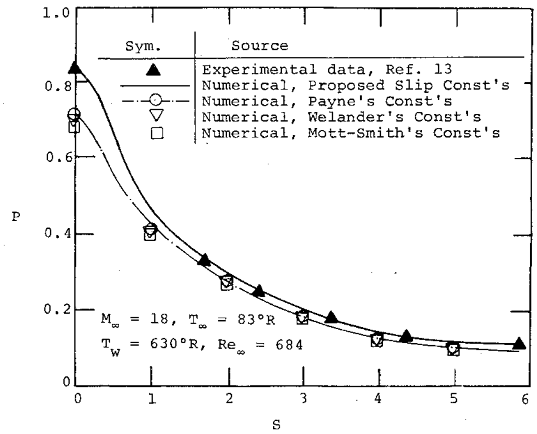

Fig. 2 Comparison of various slip constants.

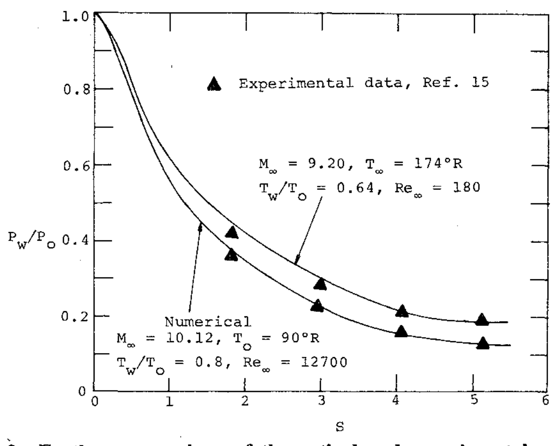

Fig. 3 Further comparison of theoretical and experimental surface pressure distributions.

dition. This indicated that the jump conditions at the surface might not be represented correctly with presently available models. Particular interest centered on various slip constants used in the preceding set of calculations, given as

$$a_I = 1.2304(2-\theta_r)/\theta_r$$

$$b_I = 1.1750(2-\theta_r)/\theta_r$$

$$c_I = 2.3071(2-\alpha_t)/\alpha_t$$

where θ_r = fraction of incident molecules diffusely reflected and α_t = thermal accommodation coefficient.

The possibility that the slip constants a_I, b_I, c_I did not represent the present flow correctly was explored in a numerical experiment to assess the sensitivity of the results to those values. Since the Stanton number was, in general, in close agreement with experiment, only the constants appearing in a_I and b_I (which directly influence the pressure level) were varied in the study. New constants were established which exactly correspond to a 40% increase in θ_r. However, since θ_r by nature should be unity or less, this result translates to the new set of coefficients, where the constants 1.2304 in a_I are replaced by 0.527 and 1.1750 in b_I by 0.505. Figure 1 shows excellent comparison of the experimental and numerical pressure distributions using the foregoing set of slip constants. It has been found that the change in the coefficients of the slip constants did not produce any loss in the good comparisons of the Stanton number.

Note is made here of the fact that there have been many efforts in the past to determine these constants by other authors. Figure 2 shows the predicted numerical results obtained here using the slip constants proposed by Payne,[5] Welander,[6] and Mott-Smith[4] in comparison with those established in the present analysis. Although all of these slip constants seem to yield the same results for downstream points, the slip constants in the present form show the correct trend of the experimental data at the stagnation point also.

Further comparisons were made with Little's data[15] for flow past a paraboloid over a range of freestream Reynolds numbers of 180 to 12,700 and a freestream Mach number of 10. The predicted results, shown in Fig. 3, compare well with the experimental data.

Based on the comparisons between the theoretical calculations and the experimental data presented here, it appears that the viscous shock layer model is accurate over the major portion of a blunt surface for a wide range of Reynolds numbers. The apparent discrepancy in the predicted surface pressure in the stagnation region apparently is attributable to the slip constants used to represent the higher-order Reynolds number effects at the surface. The constants established in this study seem to correct this deficiency.

References

[1] Cheng, H. K., "The Blunt-Body Problem in Hypersonic Flow at Low Reynolds Number," Rept. AF-1285-A-10, June 1963, Cornell Aeronautical Laboratory, Buffalo, N. Y.

[2] Jain, A. C. and Adimurthy, V., "Hypersonic Merged Stagnation Shock Layers. Part I. Adaibatic Wall Case," *AIAA Journal*, Vol. 12, March 1974, pp. 342-347.

[3] Sherman, F. A., "A Low Density Wind-Tunnel Study of Shock Wave Structure and Relaxation Phenomena in Gases," TN 3298, 1955, NACA.

[4] Mott-Smith, H. M., "A New Approach in the Kinetic Theory of Gases," Group Rept., V-2, Dec. 1954, Lincoln Laboratory, Massachusetts Institute of Technology, Cambridge, Mass.

[5] Payne, M., "Temperature-Jump and Velocity of Slip at the Boundary of Gas," *Journal of Chemical Physics*, Vol. 21, 1953, pp. 2127-2131.

[6] Welander, P., "On Temperature Jump in Rarefied Gas," *Arkiv fur Fysik*, Vol. 7, 1954, pp. 507-553.

[7] Cheng, S.-I, and Chen, J. H., "Slip, Friction and Heat Transfer Laws in a Merged Regime," *Physics of Fluids*, Vol. 17, Sept. 1974, pp. 1677-1687.

[8] Davis, R. T., "Numerical Solution of the Hypersonic Viscous Shock Layer Equations," *AIAA Journal*, Vol. 8, May 1970, pp. 843-851.

[9] Thyagaraja, A., "Compressible Flows at Small Reynolds Numbers," Ph. D. thesis, 1972, Department of Mathematics, California Institute of Technology, Pasadena, Calif.

[10] Blottner, F. G. and Flugge-Lotz, I., "Finite-Difference Computation of the Boundary Layer with Displacement Interaction," *Journal de Mecanque*, Vol. II, April 1963.

[11] Srivastava, B. N., Werle, M. J., and Davis, R. T., "Solution of the Hypersonic Viscous Shock Layer Equations for Flow Past a Paraboloid," Rept. AFL-74-4-10, April 1974, Department of Aerospace Engineering, University of Cincinnati, Cincinnati, Ohio.

[12] Davis, R. T. and Nei, Y. W., "Numerical Solution of the Viscous Shock Layer Equations for Flow Past Spheres and Paraboloids," Final Rept., Contract 48-9195, Sandia Corporation.

[13] Boylan, D. E., private communication, Aug. 1971, Aerodynamics Division, Von Karman Gas Dynamics Facility, Arnold Rsearch Organization, Arnold Air Force Station, Tenn.

[14] Davis, R. T., "Hypersonic Flow of a Chemically Reacting Binary Mixture Past a Blunt Body," AIAA paper 70-805, Los Angeles, Calif., 1970.

[15] Little, H. R., "Surface Conditions on Hyperboloids and Paraboliods in Hypersonic Flow," AIAA paper, 70-182, New York, N. Y. 1970.

Disjoint Design Spaces in the Optimization of Harmonically Excited Structures

Erwin H. Johnson*
NASA Ames Research Center, Moffett Field, Calif.

Nomenclature

G	= shear modulus (psi)
$I_{\alpha 0}$	= mass polar moment of inertia/thickness (slugs)
J_0	= area polar moment of inertia/thickness (in.^{3})
L	= length of the rod (in.)
R	= radius of the rod (in.)
S	= nondimensional spanwise coordinate
t_i	= thickness, design variable (in.)
$\bar{T}$	= amplitude of the applied torque (lb)
θ	= rotational displacement (rad)
λ	= nondimensional frequency ($= \omega^2 I_{\alpha 0} L^2 / (6GJ_0)$)
ω_e	= frequency of the harmonic excitation (rad/sec)

THE optimization of structures undergoing harmonic excitation serves as a valuable starting point for structural optimization studies with more general dynamic loadings. References 1-3 deal with the minimum weight design of one-dimensional structures under such conditions by developing optimality criteria based on energy methods. In these studies, equality constraints were placed on the amplitude of the displacement or on the dynamic compliance. It was also necessary to specify that the structure's first natural frequency of vibration be greater than the excitation frequency. This

Received July 11, 1975; revision received September 29, 1975. The author gratefully acknowledges the fellowship support he received from the Fannie and John Hertz Foundation while performing the research reported on in this Note.

Index categories: Structural Design, Optimal; Structural Dynamic Analysis.

*National Research Council Postdoctoral Fellow. Member AIAA.